WIENER NUMBER OF SOME EDGE DELETED GRAPHS

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Abstract

Let G be a connected graph. If $u, v \in V\,(G)$, then $d(u, v)$ is the length of the shortest $u - v$ path in G. The wiener number $W(G)$ of a graph G is defined as $\mathrm{W}(G) = \frac{1}{2}\sum_{u,v\in V(G)} d(u, v)$ where the summation extends over all possible pairs of distinct vertices u and v in $V(G)$. In this paper, we determine the wiener number for graphs obtained from a complete graph by deleting some of its edges. The induced subgraph of the deleted edges form barbell, tadpole, complete bipartite, lollipop and bistar.

Keywords*:* distance, wiener number, barbell, tadpole, lollipop.

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1. Introduction

By a graph $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The *order* and *size* of G are denoted by p and q respectively. For basic definitions and terminologies we refer to [1,2,4]. For vertices u and v in a connected graph G, the distance $d(u, v)$ is the length of a shortest $u - v$ path in G. The wiener number $W(G)$ of a graph G is defined as $\mathrm{W}(G) = \frac{1}{2}\sum_{u,v\in V(G)} d(u, v)$ where the summation extends over all possible pairs of distinct vertices u and v in $V(G)$. This concept was first introduced by American chemist Harold Wiener in 1947. He used wiener number as one of the structural descriptors for acyclic organic molecules in chemistry. But, the definition of the wiener number in terms of distance between vertices of a graph was first given by Hosoya in 1969. Later, various authors studied this concept in [3]. A *clique* in a graph G is a maximal complete subgraph of G. A *complete multi-partite graph* is a graph G whose vertices can be partitioned into sets so that any two vertices u , v of G are adjacent if and only if u and v belongs to different sets of the partitions. It is denoted by $K_{n_1,n_2,\dots,n_k}$.A *Barbell graph* is the simple graph obtained by connecting two copies of a complete graph K_m by a bridge. It is denoted by B_m or $B(K_{m,m})$. The *lollipop graph* is the graph obtained by joining a complete graph K_n to a path graph P_1, with a bridge. It is denoted by $L_{m,1}$. A *friendship graph* F_m is a graph which consists of m triangles with a common vertex. A *Tadpole graph* is the graph obtained by joining the cycle C_m to a path P_1 with a bridge and is denoted by $T_{m,1}$. A *bistar* is a tree obtained from the graph K_2 with two vertices u and v by attaching m pendent edges in u and m pendent edges in v and is denoted by $B_{m,m}$. Throughout this paper G denotes a connected graph with at least two vertices.

2. The wiener number of some edge deleted graphs

Definition 2.1. Let G be a complete graph of order n. Let K_{m_1,m_2} be a complete bipartite subgraph of K_n on $m_1 + m_2$ vertices. Then $G(n, m_1, m_2)$ is the graph obtained from K_n by deleting the edges of K_{m_1,m_2}.

Theorem 2.2. $W\ (G\ (n, m_1, m_2)) = \frac{n(n-1)}{2} + m_1 m_2\ , n \geq 4.$

Proof. Consider the complete graph $G = K_n$. Delete the edges of a complete bipartite subgraph from K_n, then the resultant graph $G\ (n, m_1, m_2)$ contains m_1 vertices which are at a distance two with m_2 vertices and at a distance one with $n - m_2 - 1$ vertices. There are m_2 vertices which are at a distance two with m_1 vertices and at a distance one with $n - m_1 - 1$ vertices. The remaining $n - m_1 - m_2$ vertices which are at a distance one with $n - 1$ vertices.

Therefore, $W\ (G\ (n, m_1, m_2)) = \frac{1}{2}\ [\ m_1\ [2(\ m_2) + 1\ (n - m_2 - 1)] + m_2\ [2(\ m_1) + 1(n - m_1 - 1)] + (n - m_1 - m_2)\ (1)\ (n - 1)\ ] = \frac{1}{2}[2m_1m_2 - nm_1 - m_1m_2 - m_1 + 2m_1m_2 + nm_2 - m_1m_2 - m_2 + n^2 - n - m_1n - m_2n + m_1 + m_2\ ] = \frac{n(n-1)}{2} + m_1m_2$

Definition 2.3. Let K_n be a complete graph of order n. Let F_m be a friendship subgraph of K_n on $2m + 1$ vertices. Then $G(n, m)$ is the graph obtained from K_n by deleting the edges of F_m.

Theorem 2.4. $W\ (G\ (n, m)) = \frac{n(n-1)}{2} + 3m\ , n > 5.$

Proof. Consider the complete graph $G = K_n$. If the edges of a friendship subgraph are deleted from K_n, then the resultant graph $G\ (n, m)$ contains a vertex which is at a distance two with $2m$ vertices and at a distance one with $n - 2m - 1$ vertices. Also, there are $2m$ vertices which are at a distance two with two vertices and at a distance one with $n - 3$ vertices. And the remaining $n - 2m - 1$ vertices which are at a distance one with $n - 1$ vertices. Therefore, $W\big(G(n,m)\big) = \frac{1}{2}\big[1\,[2(2m) + 1\,(n - m - 1)] + 2m\,[2(2) + 1(n - 3)] + (n - 2m - 1)(1)(n - 1)\big]$ $= \frac{1}{2}[4m + n - 2m - 1 + 8m + 2mn - 6m + n^2 - n - 2mn + 2m - n + 1]$. Hence, $W\,(G\,(n,m)) = \frac{n(n-1)}{2} + 3m$.

Theorem 2.5. Let K_n be a complete graph of order n. Let $T_{m\,,1}$ be a tadpole subgraph on $m + 1$ vertices of K_n. Let $G(n, m + 1)$ be the graph obtained from K_n by deleting the edges of $T_{m\,,1}$. Then for $n > 5$, $W\,(G\,(n,m\, + 1)) = \frac{n(n-1)}{2} + m + 1$.

Proof. Consider the complete graph $G = K_n$**.** If the edges of a tadpole subgraph are deleted from K_n, the resultant graph $G\,(n, m + 1)$ contains a vertex which is at a distance two with three vertices and at a distance one with $n - 4$ vertices. There is a vertex (pendent vertex) which is at a distance two with a vertex and at a distance one with $n - 2$ vertices. There are $m - 1$ vertices which are at a distance 2 with 2 vertices and at a distance 1 with $n - 3$ vertices. And the remaining $n - m - 1$ vertices which are at a distance 1 with $n - 1$ vertices.

$$W\,\big(G\,(n,m + 1)\big) = \frac{1}{2}\,[1\,[2(3) + 1\,(n - 4)] + 1\,[2(1) + 1(n - 2)] + (m - 1)\;[2(2) + 1(n - 3)] + (n - m - 1)\,(1)\,(n - 1)]$$

$$W\,(G\,(n,m + 1)) = \frac{n(n-1)}{2} + m + 1$$

Definition 2.6. Let K_n be a complete graph of order n. Let $L_{m\,,1}$ be a lollipop subgraph on $m + 1$ vertices. Let $G(n, m\,,1)$be the graph obtained from K_n by deleting the edges of $L_{m\,,1}$.

Theorem 2.7. $W\,(G\,(n,m,1)) = \frac{n(n-1)}{2} + \frac{m(m-1)}{2} + 1, n > 5$

Proof. Consider the complete graph $G = K_n$. Delete the edges of a lollipop subgraph from K_n.Then the resultant graph $G\,(n, m, 1)$, there is 1 vertex which is at a distance 2 with m vertices and at a distance 1 with $n - m - 1$ vertices.There is 1 vertex (pendent vertex) which is at a distance 2 with 1 vertices and at a distance 1 with $n - 2$ vertices. There are

$m -$ 1 vertices which are at a distance 2 with $m -$ 1 vertices and at a distance 1 with $n - m$ vertices.And the remaining $n - m -$ 1 vertices which are at a distance 1 with $n -$ 1 vertices.

$$W\ (G\ (n, m, 1)) = \frac{1}{2}[1\ [2(m)\ +\ 1\ (n - m - 1)]\ +\ 1\ [2(1)\ +\ 1(n - 2)]\ + (m - 1)\ [2(m - 1) + 1(n - m)]\ +\ (n - m - 1)\ (1)\ (n - 1)$$

$$W\ (G\ (n, m, 1)) = \frac{n(n-1)}{2} + \frac{m(m-1)}{2}\ + 1$$

Definition 2.8. Let K_n be a complete graph of order n. Let B_m be a barbell subgraph on $2m$ vertices. Let $G(n, 2m)$be the graph obtained from K_n by deleting the edges of B_m.

Theorem 2.9. $W\ (G\ (n, 2m))\ = \frac{n(n-1)}{2} +\ m^2 + m + 1, n > 5$

Proof. Consider the complete graph $G = K_n$. If the edges of a barbell subgraph are deleted from K_n, the resultant graph $G(n, 2m)$, there are 2 vertices which are at a distance 2 with m vertices and at a distance 1 with $n - m -$ 1 vertices. There are $2m -$ 2 vertices which are at a distance 2 with $m -$ 1 vertices and at a distance 1 with $n - m$ vertices. The remaining $n - 2m$ vertices which are at a distance 1 with $n -$ 1 vertices.

Therefore, $W\ (G\ (n, 2m)) = \frac{1}{2}[2\ [2(m)\ +\ 1\ (n - m - 1)]\ +\ 2m - 2\ [2(m - 1)\ + 1(n - m)] + (n - 2m)(n - 1)]$

$$W\ (G\ (n, 2m))\ = \frac{n(n-1)}{2} + m^2 -\ m\ +\ 1$$

Definition 2.10. Let K_n be a complete graph of order n. Let $B_{m.m}$ be a bistar subgraph on$2m + 2$ vertices. $G(n, m, m)$be the graph obtained from K_n by deleting the edges of $B_{m,m}$.

Theorem 2.11. $W(G(n, m, m))\ = \frac{n(n-1)}{2} + 2m\ +\ 1, n \geq 4$

Proof. Consider the complete graph $G = K_n$. Delete the edges of a bistar subgraphs from K_n. The resultant graph $G(n, m, m)$ there are 2 vertices which are at a distance 2 with $m\ +\ 1$ vertices and distance 1 with $n - m - 2$ vertices. There are $2m$ vertices which are at a distance 2 with 1 vertex and distance 1 with $n - 2$ vertices.And the remaining $n - 2m - 2$ vertices which are at a distance 1 with $n -$ 1vertices. $W(G(n, m, m) = \frac{1}{2}[2\ [2(m + 1)\ +\ 1\ (n - m - 2)]\ +\ 2m\ [2(1)\ +\ 1(n - 1)]\ (n - 2m - 2)(1)(n - 1)$

$$W\ (G\ (n, m, m)) = \frac{n(n-1)}{2} + 2m\ +\ 1$$

Definition 2.11. Let K_n be a complete graph of order n. Let Sp_m be a spider subgraph on $2m + 1$ vertices . $G(n, m)$ be the graph obtained fromK_n by deleting the edges of Sp_m.

Theorem 2.12. $W(G(n, m)) = \frac{n(n-1)}{2} + 2m, n \geq 4$

Proof. Consider the complete graph $G = K_n$. Delete the edges of a spider subgraphs from K_n. Then the resultant graph $G(n, m)$ contains only one vertex which is at a distance two with m vertices and at a distance one with $n - m - 1$. There are m vertices which are at a distance 2 with 2 vertices and distance 1 with $n - 3$ vertices.There are m vertices (pendent vertices) which are at a distance 2 with 1 vertex and distance 1 with $n - 2$ vertices.And the remaining $n - 2m - 2$ vertices which are at a distance 1 with $n - 1$ vertices. $W(G(n, m)) = \frac{1}{2}[1[2(m) + 1(n - m - 1)] + m[2(2) + 1(n - 2)] + (n - 2m - 2)(n - 1)]$ Hence, $W(G(n, m)) = \frac{n(n-1)}{2} + 2m$.

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